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ON THE INITIAL DEVELOPMENT OF SLIP LINES FROM THE FREE BOUNDARY OF A SOLID

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V. D. KULIEV and G. P. CHEREPANOV
(Moscow)

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An exact, closed solution of the elastic-plastic problem of two slip lines issuing at an arbitrary angle from a certain point of the free boundary of a half-plane subjected to tension or compression is given by the Wiener — Hopf method. A stress concentration describable by a certain stress intensity factor is allowed near the end of the slip lines. The beginning of slip line motion is described by using a theory analogous to the flow lag theory of Cottrell — Rabotnov.

One of the most interesting questions in principle, in the theory of ideal plasticity, is the question of the structure of the plastic deformations which can be concentrated in narrow Lüders — Chernov slip layers (located entirely in the elastic domain) or can be distributed continuously in some zone (whose volume is substantially different from zero). On the basis of experimental data (see [1], for instance) for the existence of slip lines, and abrupt passage from the elastic to the plastic mode is necessary on the $\sigma - \varepsilon$ diagram, as is also the presence of a quite definite flow area on the same diagram. Not only low-carbon steel, but also some other alloys possess such properties under appropriate heat treatment [1]. The presence of a quite definite flow area is, generally, substantially insufficient for the formation of slip lines. (A titanium alloy, possessing this property, is described in particular in [2]).

On the other hand, the distributed plastic zones in an ideal elastic-plastic body are transformed in the presence of small holes and notches, into long "tongues" extending deep into the body as the external load increases. This fundamental property of plastic deformations can be considered established not only experimentally, but also on the basis of a number of exact solutions of elastic-plastic problems (for instance, the Trefftz solution for continuous shear [3], the Southwell and Allen solution for plane deformation [4], and the Cherepanov solution for the plane state of stress [5]). Naturally, for a sufficiently long tongue it can be considered a certain slip line in the elastic domain, i.e., the thickness of the tongue can be neglected as compared to its length. A jump (discontinuity) in the stress and displacement, which satisfies the usual conservation laws, hence occurs on the slip line.

Reuss [6] first conceived of the need to introduce stresses exceeding the yield point in the elastic domain in a theoretical study of slip lines.

1. Plastic deformation model. Following Reuss [6], let us consider the stresses in the elastic domain to exceed the yield point. Under definite conditions the origination of a stress concentration at the end of the slip line hence follows. Indeed, let a cracklike ("thin") cavity located along the line Os of the xOy plane be filled compactly with material whose yield points is less than for the

main material. (The cavity is considered thin if the conditions $h \ll L, dh / ds \ll 1$ are satisfied, where $h(s)$ and L are the transverse and characteristic longitudinal dimensions of the cavity). The "filler" material is considered ideally plastic. As the external load increases, plastic shear will evidently occur initially in the fine cavity. The displacement compoent in the tangential plane to the surface of the crack-like cavity hence undergoes a discontinuity, while the normal displacement is continuous in this plane. The normal and tangential stresses will also be continuous (equilibrium conditions). It can moreover be assumed that the tangential stress in the slip plane is not dependent on the normal stress in this plane in the limit state. Therefore, $\tau = \tau_s$ can be taken as the condition for the limit state of a plastic material in a thin cavity, where τ_s is the shear yield point and the stress τ is considered to act on the edges of the discontinuity.

Hence, it is seen that as soon as the limit state of the plastic material is reached in the thin cavity, a stress concentration can occur near its edge, which is characterized by the elastic asymptotic for a transverse shear crack (with a plastic filler). As the external load increases further, this concentration (if such there be) will grow. It is hence assumed that the increase in the external loads would be such that the stresses in the main material would be less than its yield point $2\tau_{sb}$. In other words, the stresses σ in the elastic domain take on the values $2\tau_s \leqslant \sigma < 2\tau_{sb}$. The existence of an elastic equilibrium of the stresses for which the stresses σ "slip past" the lower yield point $2\tau_s$ without causing plastic deformation is thereby allowed because of the "thinness" of the cavity under consideration.

Therefore, the development of plastic deformation along the thin slip band surrounded by the elastic domain can be explained, exactly as by Reuss, by the inhomogeneity with respect to plastic deformations inherent in the material beforehand.

The condition for the occurrence of a stress concentration in a small domain at the end of the slip line depends, within the framework of this model, on the magnitude of the slip viscosity, and thereby, on the strucutre and strength of the material of this domain.

As is shown above, the stress concentration at the end of the slip line is actually characterized in the elastic domain by the elastic asymptotic for transverse shear crack. This asymptotic (local for hyperthin) is determined fully by one stress intensity factor K_{II}. Let K_{IIc} be the slip viscosity for a transverse shear crack without a filler, and K_{IIcb} the slip viscosity for cracks with a plastic filler. From physical considerations $K_{IIc} < K_{IIcb}$. As the external load increases monotonically, the coefficient K_{II} will evidently also grow monotonically and slip past the lower limit of the slip viscosity K_{IIc} without causing any plastic deformation at the end of the slip line in the elastic domain during a certain time, i.e., when the condition $0 \leqslant K_{IIc} < K_{II} < K_{IIcb}$. When $K_{II} = K_{IIcb}$, plastic deformation suddenly appears at the end of the slip line in the elastic domain and motion of the slip line starts, where $K_{II} = K_{IIc}$ during motion.

The slip viscosity K_{IIcb} determines the resistance of the material to the development of slip surfaces and is a constant of the material. When such a resistance is negligibly small, it can be considered that $K_{IIcb} = 0$, and the stresses at the end of the slip line will be bounded only in this particular case.

The plastic deformation model presented is characteristic for media with a flow delay, and consequently, just such media will be examined in this paper.

Realization of any of the possible solutions in practice (for example, the discontinuous and continuous as in these problems) can be explained from the aspect of the general selection principle formulated by Cherepanov [7], and which is an integral generalization of the Mises maximum principle [8].

Let us note some of the solutions with plastic slip lines which are available in the literature: The Dugdale solution [9] (a slip line on the continuation of a crack in a plate), the solution of M. Ia. Leonov and his colleagues [10] (for different cases of rod torsion and the extension of thin plates with stress concentrators), the Cherepanov solution [11] (one slip line emerging on the free boundary of a half-plane) etc.

Within the framework of the model presented, an exact solution of the problem of two slip lines emerging at an arbitrary angle from a certain point of the free boundary of a half-plane subjected to tension of compression is constructed below. A local stress concentration described by a certain stress intensity factor is allowed at the "head" of the slip line in the elastic domain. The beginning of slip line motion (i. e., the beginning of the origination of plastic deformations at the head of the slip line in the elastic domain) is described within the framework of a theory (*), analogous to the flow lag theory of Cottrell — Rabotnov [12, 13], however, the formulated in terms of the stress intensity factor (instead of the stress). It is found that the slip lines should make a 45° angle with the free boundary of the body. Such a slip line configuration is observed under plane strain conditions if the site of their origination is fixed in advance by using a shallow groove or notch (see [1], for instance).

2. Boundary value problem. Let a homogeneous and isotropic elastic body occupy the half-plane $x > 0$ in the Oxy plane, where xy are rectilinear Cartesian coordinates. Rectilinear slip lines of length l will emerge at an angle 2α, symmetrically relative to the x axis on the boundary of the half-plane which is free of external loads (Fig. 1). Without limiting the generality, the length l can be considered one (the characteristic length scale). We shall henceforth use the polar coordinates $r\theta$ with center at the origin of the Cartesian coordinates. Let the constant stress $\sigma_y = \sigma$ act at infinity.

We write the boundary conditions of a somewhat different problem thus:

$$\theta = 0,\ \tau_{r\theta} = 0,\ u_\theta = 0 \tag{2.1}$$

$$\theta = \pi / 2,\ \sigma_\theta = \tau_{r\theta} = 0$$

$$\theta = \alpha,\ [\sigma_\theta] = [\tau_{r\theta}] = 0,\ [u_\theta] = 0$$

$$\theta = \alpha,\ \tau_{r\theta} = \tau_s - 1/2\,\sigma \sin 2\alpha\ (0 < r < 1),\ \ [\sigma_r] = 0\ \ (r > 1) \tag{2.2}$$

Here σ_θ, σ_r, $\tau_{r\theta}$ are the stresses, and u_θ, u_r the displacements. The square brackets denote jumps in the quantities enclosed by the brackets. It is assumed that $\sigma \geqslant 2\tau_s$.

Let the condition

*) Let us note that in essence only the purely physical qualitative aspect of the theory is considered in [12]; the complete three-dimensional mathematical theory is developed in [13].

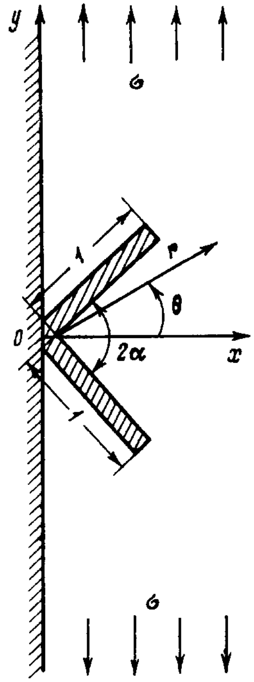

Fig. 1

$$r \to \infty, \ \sigma_x = \sigma_y = \tau_{xy} = 0 \tag{2.3}$$

be satisfied at infinity.

The stresses in the initial problem shown in Fig. 1 are evidently equal to the stresses and strains obtained from the solution of the boundary value problem (2. 1)–(2. 3) plus the stresses $\sigma_x = \tau_{xy} = 0, \ \sigma_y = \sigma$.

Because of symmetry relative to the x axis, it is sufficient to construct the solution of the problem for $\pi / 2 > \theta > 0$.

3. **Derivation of the Wiener – Hopf equation.** Applying the integral transformation (p is a complex parameter)

$$\varphi^*(p) = \int_0^\infty \varphi(r) r^p \, dr \tag{3.1}$$

to the equilibrium equations and the strain compatibility condition, we obtain

$$\tau_{r\theta}^* = \frac{1}{p-1} \frac{d\sigma_\theta^*}{d\theta}, \quad p\sigma_r^* = \frac{1}{p-1} \frac{d^2\sigma_\theta^*}{d\theta^2} - \sigma_\theta^* \tag{3.2}$$

$$\sigma_\theta^* = \begin{cases} \sigma_\theta^{*+}, & 0 \leqslant \theta < \alpha \\ \sigma_\theta^{*-}, & \alpha < \theta \leqslant \pi/2 \end{cases} \tag{3.3}$$

$$\sigma_\theta{}^{*+} = A_1 \cos(p+1)\theta + A_2 \cos(p-1)\theta + A_3 \sin(p+1)\theta + A_4 \sin(p-1)\theta \tag{3.4}$$

$$\sigma_\theta{}^{*-} = B_1 \cos(p+1)(\pi/2 - \theta) + B_4 \cos(p-1)(\pi/2 - \theta) + B_2 \sin(p+1)(\pi/2 - \theta) + B_3 \sin(p-1)(\pi/2 - \theta)$$

Here $A_i, \ B_i \ (i = 1, 2, 3, 4)$ are unknown functions of the complex parameter p to be determined from the transformed boundary conditions. Any seven of them are expressed in terms of one unknown function by using seven "through" conditions (2. 1), transformed in r.

By using (3. 2) –(3. 4) and Hooke's law, we arrive from the transformed boundary conditions (2. 1) –(2. 2) to the following system of equations

$$(p+1) A_3 + (p-1) A_4 = 0 \tag{3.5}$$

$$A_3 [(p+1) - 4(1-\nu)] + (p-1) A_4 = 0$$

$$B_1 + B_4 = 0, \ B_2 (p+1) + B_3 (p-1) = 0$$

$$A_1 (p-1) \cos(p+1)\alpha + A_2 (p-1) \cos(p-1)\alpha = 2 B_2 [p \cos p(\pi/2 - \alpha) \cos\alpha - \sin p(\pi/2 - \alpha) \sin\alpha] - 2 B_1 (p-1) \sin p(\pi/2 - \alpha) \cos\alpha$$

$$A_1 (p+1) \sin(p+1)\alpha + A_2 (p-1) \sin(p-1)\alpha = -2 B_2 (p+1) \sin p(\pi/2 - \alpha) \cos\alpha - 2 B_1 \times$$

$$[p \cos p(\pi / 2 - \alpha) \cos \alpha + \sin p (\pi / 2 - \alpha) \sin \alpha]$$
$$A_1 \sin (p + 1) \alpha = B_2 \cos (p + 1) (\pi / 2 - \alpha) - B_1 \sin (p + 1) (\pi / 2 - \alpha)$$

We write the solution of the system (3. 5) in the form

$$A_1 = C [p \cos \alpha \sin p\alpha + \sin p (\pi / 2 - \alpha) \cos (p\pi / 2 - \alpha)] \tag{3.6}$$
$$A_2 = \frac{C}{p-1} \left[p^2 \cos \alpha \sin p\alpha + \sin p \left(\frac{\pi}{2} - \alpha\right) \cos \left(p \frac{\pi}{2} + \alpha\right) - p \sin p \frac{\pi}{2} \cos \left(p \frac{\pi}{2} - p\alpha + \alpha\right) \right], \quad A_3 = A_4 = 0$$
$$B_1 = - B_4 = - C (p \cos \alpha \sin p\alpha + \sin \alpha \cos p\alpha) \sin p\pi / 2$$
$$B_2 = C\Delta, \; B_3 = - C (p + 1) (p - 1)^{-1} \Delta$$
$$\Delta = (p - 1) \cos \alpha \sin p\alpha \cos p\pi / 2$$

Let us introduce the following functions

$$\Psi^- (p) = \int_0^1 [\sigma_r] |_{\theta=\alpha} r^p \, dr, \quad \Psi^+ (p) = \int_1^\infty \tau_{r\theta} (r, \alpha) r^p dr$$

The function $\Psi^- (p)$ is analytic in the half-plane $\operatorname{Re} p > -1$, while the functions $\Psi^+ (p)$ is analytic in the half-plane $\operatorname{Re} p < 1$.

By using the functions introduced, we can write conditions (2. 2) as

$$\theta = \alpha, \; [\sigma_r^*] = \Psi^- (p), \; \tau_{r\theta}^* (p, \alpha) = \Psi^+ (p) + F (p) \tag{3.7}$$
$$F (p) = \int_0^1 \left[\tau_s - \frac{1}{2} \sigma \sin 2\alpha \right] r^p dr$$

By using (3. 2), (3. 4), (3. 6), we obtain

$$\Psi^+ - (p) + F (p) = - \frac{C}{p-1} \gamma (p) \tag{3.8}$$
$$\Psi^- (p) = \frac{2C}{p-1} \sin p\pi$$
$$\gamma (p) = 2 p^2 \cos^2 \alpha \sin^2 p\alpha + \sin p (\pi / 2 + \alpha) \times \sin p(\pi / 2 - \alpha) - \sin^2 p (\pi / 2 - \alpha) \cos 2 \alpha + p \sin 2 \alpha \times \sin p\pi / 2 \cos p (\pi / 2 - 2 \alpha)$$

Eliminating the functions $C (p)$ from the two relationships in (3. 8), we arrive at the functional Wiener — Hopf equation

$$\Psi^+ (p) + F (p) = 1/4 \operatorname{ctg} p\pi G (p) \Psi^- (p) \tag{3.9}$$
$$G (p) = 1 - \frac{1}{\cos p\pi} \left[4p^2 \cos^2 \alpha \sin^2 p\alpha + \cos 2p\alpha + 2p \sin 2\alpha \sin p\frac{\pi}{2} \cos p \left(\frac{\pi}{2} - 2\alpha\right) - 2 \cos 2\alpha \sin^2 p \left(\frac{\pi}{2} - \alpha\right) \right]$$

4. **Solution of the Wiener — Hopf equation.** The functional equation (3.9) holds in the strip $|\operatorname{Re} p| < 1$. The function $G(p)$ possesses the following properties:

a) The function $G(p)$ is meromorphic, where all the poles located at the points $p = \pm 1/2 \pm n$ $(n = 1, 2, 3, \ldots)$, are simple;

b) The function $G(p)$ has neither poles nor zeros anywhere on the imaginary axis with the exception of the point $p = 0$ where it has a second order zero, where

$$G(p) = p^2 [2(\alpha^2 - \pi^2/4) - \pi \sin 2\alpha + 2(\pi/2 - \alpha)^2 \cos 2\alpha] + O(p^3) \quad (p \to 0)$$

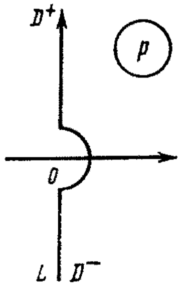

Fig. 2

c) as $p \to \infty$ along the imaginary axis $G(p) \to 1$ because of the inequality $\alpha < \pi/2$.

Let us consider the contour L consisting of the imaginary axis (with the exception of a small symmetric segment around the origin) and a right semi-circle of small radius with center at the origin (Fig. 2) in the p plane. The direction of traversing the contour agrees with the direction of the imaginary axis. We denote the domains to the left and right of the contour L by D^+ and D^-, respectively.

The function $G(p)$ can be represented in the form

$$G(p) = G^+(p) / G^-(p) \quad (p \in L) \tag{4.1}$$

$$\exp \frac{1}{2\pi i} \int_L \frac{\ln G(t)}{t - p} dt = \begin{cases} G^+(p) & (p \in D^+) \\ G^-(p) & (p \in D^-) \end{cases} \tag{4.2}$$

The functions $G^+(p)$ and $G^-(p)$ are analytic and have no zeroes in the domains D^+ and D^-, respectively; they tend to one at infinity.

Let us use the following known representation (see [14, 15], for example):

$$p \operatorname{ctg} p\pi = K^+(p) K^-(p) \tag{4.3}$$

$$K^\pm(p) = \Gamma(1 \mp p) / \Gamma(1/2 \mp p)$$

According to the properties of Gamma functions, the function $K^+(p)$ is analytic and has no zeroes for $\operatorname{Re} p < 1/2$, while the function $K^-(p)$ is analytic and has no zeroes for $\operatorname{Re} p > -1/2$. Moreover, according to the Stirling formula we have

$$K^\pm(p) = \sqrt{\mp p} + O(1) \quad (p \to \infty) \tag{4.4}$$

Taking the factorization of (4.1), (4.3) into account, (3.9) can be written thus

$$\frac{\Psi^+(p)}{K^+(p) G^+(p)} + \frac{F(p)}{K^+(p) G^+(p)} = \frac{K^-(p)}{4pG^-(p)} \Psi^-(p) \quad (p \in L) \tag{4.5}$$

Now, let us use the following representation

$$\frac{F(p)}{K^+(p) G^+(p)} = F^+(p) - F^-(p) \quad (p \in L) \tag{4.6}$$

$$\frac{1}{2\pi i} \int_L \frac{F(t)}{K^+(t) G^+(t)} \frac{dt}{t - p} = \begin{cases} F^+(p) & (p \in D^+) \\ F^-(p) & (p \in D^-) \end{cases}$$

Substituting (4.6) into (4.5), we obtain

$$\frac{\Psi^+(p)}{K^+(p)\,G^+(p)} + F^+(p) = \frac{K^-(p)\,\Psi^-(p)}{4pG^-(p)} + F^-(p) \tag{4.7}$$

The left side of this equation is analytic in D^+, and the right side in D^-. On the basis of the principle of continuous extension, they equal the same function analytic in the whole p plane. In order to find this single analytic function, the behavior of the desired functions $\Psi^+(p)$ and $\Psi^-(p)$ must be studied at infinity as $p \to \infty$. This behavior is determined from the known asymptotic near the end of the slip line for $\theta = \alpha$, $r \to 1$ (see [16], p. 75)

$$\Psi^-(p) = -\frac{2\sqrt{2}K_{II}}{\sqrt{p}}, \quad \Psi^+(p) = \frac{K_{II}}{\sqrt{-2p}}, \quad p \to \infty \tag{4.8}$$

On the basis of (4.2), (4.4) and (4.8), the single analytic function in (4.7) tends to zero as $p \to \infty$. Therefore, by the Liouville theorem it equals zero indentically in the whole p plane. Hence, the solution of the Wiener — Hopf equation has the form

$$\Psi^+(p) = -F^+(p)\,K^+(p)\,G^+(p) \tag{4.9}$$

$$\Psi^-(p) = -\frac{4pF^-(p)\,G^-(p)}{K^-(p)} \tag{4.10}$$

Hence, determining the function $C(p)$ by using (3.8), we find the Mellin transform of the desired stresses, and the stresses themselves after inverting the transforms.

5. **A n a l y s i s o f t h e s o l u t i o n.** Let us find the stress intensity factor K_{II} at the vertex of the slip line for $\theta = \alpha$. Using (4.8), (4.9), (4.2) and (4.4) as $p \to \infty$, we find

$$K_{II} = -\frac{1}{\sqrt{2}}\frac{1}{\pi i}\int_L \frac{F(t)}{K^+(t)\,G^+(t)}\,dt \tag{5.1}$$

According to (3.7), the function $F(p)$ will evidently have the form

$$F(p) = \frac{1}{p+1}\left(\tau_s - \frac{\sigma}{2}\sin 2\alpha\right) \tag{5.2}$$

Substituting this expression into (5.1) and evaluating the integral by using residue theory, we find (the formula is written in dimensional parameters)

$$K_{II} = -\sqrt{\frac{\pi}{2}}\,\frac{\tau_s - {}^1/_2\,\sigma\sin 2\alpha}{G^+(-1)}\sqrt{l} \tag{5.3}$$

Transforming the formula (4.2), we find

$$G^+(-1) = \exp\frac{1}{\pi}\int_0^\infty \frac{\ln m(t)}{1+t^2}\,dt$$

$$m(t) = 1 - \frac{1}{\operatorname{ch} t\pi}\left[4t^2\cos^2\alpha\,\operatorname{sh}^2 t\alpha + \operatorname{ch} 2t\alpha - \right.$$

$$2t \sin 2\alpha \operatorname{sh} t \frac{\pi}{2} \operatorname{ch} t \left(\frac{\pi}{2} - 2\alpha\right) + 2 \cos 2\alpha \operatorname{sh}^2 t \left(\frac{\pi}{2} - \alpha\right)\Big]$$

A graph of the dependence of $K_{II} / (\sigma\sqrt{l})$ on the angle α is constructed on an electronic computer for $\tau_s = 0$ and is presented in Fig. 3. It is seen that the quantity $K_{II} / (\sigma\sqrt{l})$ reaches the maximum value 0.86 for $\alpha = 45°$.

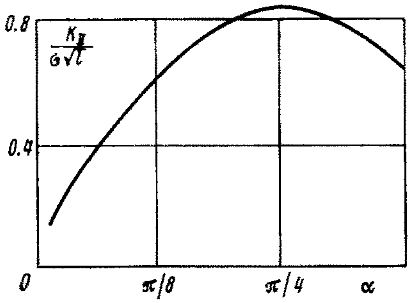

Fig. 3

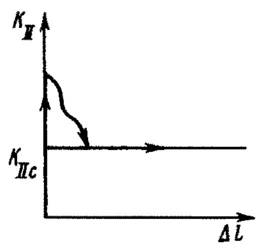

Fig. 4

Let us find the stress as $r \to 0$ and $0 < \theta < \alpha$ in the initial problem displayed in Fig. 1. After some calculations we obtain

$$\sigma_\theta = \frac{2\tau_s}{\sin 2\alpha} \cos^2\theta, \quad \sigma_r = \frac{2\tau_s}{\sin 2\alpha} \sin^2\theta, \quad \tau_{r\theta} = \frac{\tau_s}{\sin 2\alpha} \sin 2\theta$$

The factor K_{II} in the initial problem will evidently be determined by the same formula (5.3).

6. Slip line motion. It is natural to assume that the slip line will be developed in that direction α where the maximum value of the coefficient K_{II} is achieved, i.e., at $\alpha = 45°$ according to Fig. 3. This result is in good agreement with experimental data.

Let us now study slip line motion for a monotonic increase in the intensity factor K_{II}. We assume:

a) If the stress intensity factor K_{II} is less than a certain constant of the material K_{IIc}, then no slip line motion will occur for a monotonic increase in K_{II};

b) If the stress intensity factor K_{II} is greater than a certain constant of the material K_{IIc}, then slip line motion starts as time τ elapses from the beginning of the loading (for $K_{II} = 0$), where the time τ is determined by the following lag condition:

$$\frac{1}{t_0} \int_0^\tau f\left(\frac{K_{II}}{K_{II\,c}}\right) dt = 1 \tag{6.1}$$

where t_0 is some time constant, and $f(x)$ is a certain monotonically increasing function determined from test;

c) The stress intensity factor K_{II} equals the constant of the material K_{IIc} for slip line motion.

The model presented is displayed in the form of a $K_{II} - \Delta l$ diagram in Fig. 4, where Δl is an increment in the slip line length.

This model is analogous to the Clark — Cottrell — Rabotnov flow lag model for the usual specimens with a $\sigma - \varepsilon$ diagram if the analogy $K_{II} \leftrightarrow \sigma$, $\Delta l \leftrightarrow \varepsilon$ is taken into account (see [16]).

Let us take the function $f(x)$ in the following form [13]:

$$f(x) = x^n \tag{6.2}$$

where n is some constant determined from experiment.

According to (5.3), (6.1) and (6.2), we obtain

$$\frac{1}{t_*}\int_0^\tau \left(\frac{\sigma - 2\tau_s}{K_{II\,c}}\right)^n dt = 1, \quad t_* = t_0\left(\frac{\sqrt{\pi l}}{2\sqrt{2}}\right)^{-n}[G^+(-1)]^n \tag{6.3}$$

which agrees, to the accuracy of the notation, with the corresponding formula in the theory of Rabotnov [13] governing the time for plastic deformations to originate in a rod extended by the stress σ.

Therefore, this structural model yields an additional justification for the Rabotnov phenomenological theory.

Let us determine the time τ of the beginning of motion for the following two modes of increasing the load σ:

a) The load is elevated instantly to the magnitude σ and then remains invariant with the lapse of time. In this case, we have on the basis of (6.3):

$$\tau = t_*\left(\frac{K_{II\,c}}{\sigma - 2\tau_s}\right)^n \quad (\sigma \geqslant 2\tau_s)$$

b) The load is increased at the constant rate a, i.e., $\sigma = 2\tau_s + at$. In this case, we have on the basis of (6.3):

$$\tau = \left[\frac{t_*(n+1)}{a^n} K_{II\,c}^n\right]^{1/(n+1)}$$

Therefore, the time τ diminishes tending to zero, as the load σ or the velocity a increase, but tends to infinity for $\sigma \to 2\tau_s$ or for $a \to 0$.

These results are, at least qualitatively, meaningful.

The length of the slip line during its motion is determined from the condition $K_{II} = K_{IIc}$. Hence, we find by using (5.3).

$$\sigma = 2\tau_s + \frac{2\sqrt{2}}{\sqrt{\pi l}} G^+(-1) K_{IIc} \quad (\sigma \geqslant 2\tau_s) \tag{6.4}$$

Displayed schematically in Fig. 5 by a dashed line is the curve (6.4) of the dependence of σ on l. It evidently shows that the development of the slip line is unstable for $\sigma > 2\tau_s$. This result also agrees qualitatively with experimental data.

Let us estimate the magnitude of the factor K_{IIc} by starting from the fact that it describes the stationary development of the end of the "plastic tongue" along its x_1 axis for an infinitely slow increase in the external load (see Fig. 6 where the tip zone of the tongue developing from a shallow notch is displayed, for instance). We consider the tongue width constant, equal to h; the quantity h is of the order of the characteristic linear dimension of the groove. We find $K_{IIc} = \lambda\tau_s\sqrt{h}$ from dimensional analysis considerations (h is some dimensionless factor).

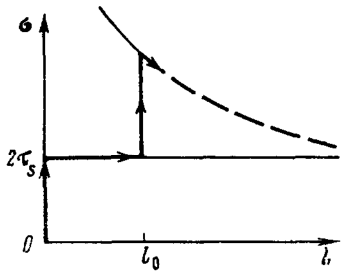

Fig. 5

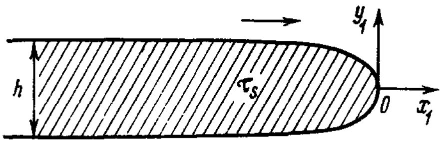

Fig. 6

For instance, if $\tau_s = 10$ kg/mm^2 and $h = 10^{-4}$ cm, then $K_{IIc} \approx 0.3$ kg/mm$^{3/2}$. This quantity is quite small: its corresponding value of the irreversible work expanded in advancing the tongue tip a single length along the x_1 axis is approximately one-twentieth of the surface energy of glass.

An analogous theory can be developed for cracks of normal discontinuity in carbon steels.

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